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Fgspr-Homeomorphism in Fuzzy Topological Spaces

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Abstract – *The purpose of this paper is to introduce a new type of fuzzy generalized homeomorphisms namely fgspr-homeomorphism and fgspr*-homeomorphism in fuzzy topological spaces and study some of their properties.*
Keywords – *fgspr-homeomorphism and fgspr*-homeomorphism.*

I. INTRODUCTION

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh [9]. Subsequently, several authors have applied various basic concepts from general topology to fuzzy sets and developed the theory of fuzzy topological spaces. Fuzzy topology and fuzzy compactness was first introduced by Chang [2]. Vadivel et al [8] introduced the concept of fuzzy generalized preregular homeomorphism and studied their properties. The concept of fgspr-closed set and fgspr-open set in fuzzy topological space was introduced by M. Thiruchelvi and Gnanambal Ilango [5]. In this paper, fgspr-homeomorphism and fgspr*-homeomorphism are introduced and their properties are studied.

II. PRELIMINARIES

Let X, Y and Z be fuzzy sets. Throughout this paper, (X, τ), (Y, σ) and (Z, η) (or simply X, Y and Z) mean fuzzy topological spaces on which no separation axioms are assumed unless explicitly stated. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function from a fuzzy topological space X to fuzzy topological space Y. Let us recall the following definitions which we shall require later.

Definition 1: A fuzzy set λ in a fuzzy topological space (X, τ) is called
(1) a fuzzy semi-preopen set [7] if $\lambda \leq cl(int(cl(\lambda)))$ and a fuzzy semi-preclosed set if $int(cl(int(\lambda))) \leq \lambda$.
(2) a fuzzy regular open set [1] if $int(cl(\lambda)) = \lambda$ and a fuzzy regular closed set if $cl(int(\lambda)) = \lambda$.

Definition 2: A fuzzy set λ in a fuzzy topological space (X, τ) is called
(1) a fuzzy generalized semi preregular closed set (briefly, fgspr-closed) [5] if $spcl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy regular open set in X.
(2) a fuzzy generalized semi preregular open set (briefly, fgspr-open) [5] if $\mu \leq spint(\lambda)$, whenever $\mu \leq \lambda$ and μ is a fuzzy regular closed set in X.

Definition 3: Let X and Y be two fuzzy topological spaces. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called
(1) a fuzzy continuous (briefly, f-continuous) [2] if $f^{-1}(\lambda)$ is a fuzzy open (fuzzy closed) set in X, for every fuzzy open (fuzzy closed) set λ in Y.
(2) a fuzzy semi pre continuous (briefly, fsp-continuous) [7] if $f^{-1}(\lambda)$ is a fuzzy semi preopen (fuzzy semi preclosed) set in X, for every fuzzy open (fuzzy closed) set λ in Y.
(3) a fuzzy generalized semi preregular continuous (briefly, fgspr-continuous) [4] if $f^{-1}(\lambda)$ is a fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set in X, for every fuzzy open (fuzzy closed) set λ in Y.
(4) a fuzzy generalized semi preregular irresolute (briefly, fgspr-irresolute) [4] if $f^{-1}(\lambda)$ is a fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set in X, for every fuzzy generalized semi preregular open (fuzzy generalized semi preregular closed) set λ in Y.
(5) a fuzzy generalized semi preregular open (briefly, fgspr-open) [6] if $f(\lambda)$ is a fgspr-open set in Y, for every fuzzy open set λ in X.
(6) a fuzzy generalized semi preregular closed (briefly, fgspr-closed) [6] if $f(\lambda)$ is a fgspr-closed set in Y, for every fuzzy closed set λ in X.
(7) a fuzzy generalized semi preregular*-open (briefly, fgspr*-open) [6] if $f(\lambda)$ is a fgspr-open set in Y, for every fgspr-open set λ in X.

(8) a fuzzy generalized semi preregular*-closed (briefly, fgspr*-closed) [6] if f(λ) is a fgspr-closed set in Y, for every fgspr-closed set λ in X.

Definition 4: [5] For any fuzzy set λ in any fuzzy topological space (X, τ),
(1) fgspr-cl(λ) = ∧ {μ: μ is a fgspr-closed set and μ ≥ λ }
(2) fgspr-int(λ) = ∨ {μ: μ is a fgspr-open set and μ ≤ λ }
Definition 5: A fuzzy topological space (X, τ) is called
(1) a fuzzy semi preregular $T_{1/2}$ space [5] if every fgspr-closed is fuzzy semi preclosed.
(2) a fuzzy semi preregular $T^*_{1/2}$ space [5] if every fgspr-closed is fuzzy closed.
Definition 6: Let X and Y be two fuzzy topological spaces. A bijective function f: (X, τ) → (Y, σ) is called
(1) a fuzzy homeomorphism (briefly, f-homeomorphism) [3] iff both f and f^{-1} are fuzzy continuous.
(2) a fuzzy semi pre homeomorphism (briefly, fsp-homeomorphism) [7] iff both f and f^{-1} are fsp-continuous.

III. FGSPR-HOMEOMORPHISM

In this section, a new type of fuzzy generalized homeomorphism called fuzzy generalized semi preregular homeomorphism is defined and its properties are studied.

Definition 7: Let X and Y be two fuzzy topological spaces. A bijective function f: (X, τ) → (Y, σ) is called fuzzy generalized semi preregular homeomorphism (briefly fgspr-homeomorphism) iff both f and f^{-1} are fgspr-continuous.

We denote the family of all fgspr-homeomorphism of a fuzzy topological space (X, τ) onto itself by FGSPR-H(X, τ).

Example 1: Let X = {a, b, c}, Y = {a, b, c} and consider the fuzzy sets λ_1 = {(a, 0.5), (b, 0.2), (c, 0.7)} and λ_2 = {(a, 0.7), (b, 1), (c, 0.5)}. Let τ = {0, λ_1, 1} and σ = {0, λ_2, 1}. Define the function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = c and f(c) = b. Then f and f^{-1} are fgspr-continuous. Therefore f is fgspr-homeomorphism.

Theorem 1: Every f-homeomorphism is fgspr-homeomorphism.
Proof: Let f: (X, τ) → (Y, σ) is f-homeomorphism. Then f and f^{-1} are f-continuous. Since every f-continuous is fgspr-continuous, therefore f and f^{-1} are fgspr-continuous. Hence f is fgspr-homeomorphism.

The following example shows that the converse of the above theorem is not true.

Example 2: Let X = {a, b, c}, Y = {a, b, c} and consider the fuzzy sets λ_1 = {(a, 0.5), (b, 0.3), (c, 0.8)} and λ_2 = {(a, 0.8), (b, 1), (c, 0.6)}. Let τ = {0, λ_1, 1} and σ = {0, λ_2, 1}. Define the function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = c and f(c) = b. Then f and f^{-1} are fgspr-continuous but are not f-continuous. Therefore f is fgspr-homeomorphism and not f-homeomorphism.

Theorem 2: If a function f: (X, τ) → (Y, σ) is fgspr-homeomorphism and X and Y are fuzzy semi preregular $T^*_{1/2}$ space then f is f-homeomorphism.
Proof: Let f: (X, τ) → (Y, σ) is fgspr-homeomorphism. Then f and f^{-1} are fgspr-continuous. To prove that f and f^{-1} are f-continuous. Let λ be a fuzzy closed set in Y. Then $f^{-1}(\lambda)$ is a fgspr-closed set in X, since f is fgspr-continuous. Also since X is fuzzy semi preregular $T^*_{1/2}$ space, $f^{-1}(\lambda)$ is a fuzzy closed set in X. Hence f is f-continuous.
Now, Let λ be a fuzzy closed set in X. Then $(f^{-1})^{-1}(\lambda)$ = f(λ) is a fgspr-closed set in Y, since f^{-1} is fgspr-continuous. Also, since Y is fuzzy semi preregular $T^*_{1/2}$ space, f(λ) is a fuzzy closed set in Y. Hence f^{-1} is f-continuous. Thus f is f-homeomorphism.

Theorem 3: Every fsp-homeomorphism is fgspr-homeomorphism.
Proof: Let f: (X, τ) → (Y, σ) is fsp-homeomorphism. Then f and f^{-1} are fsp-continuous. Since every fsp-continuous is fgspr-continuous, therefore f and f^{-1} are fgspr-continuous. Hence f is fgspr-homeomorphism.

The following example shows that the converse of the above theorem is not true.

Example 3: Let X = {a, b, c}, Y = {a, b, c} and consider the fuzzy sets λ_1 = {(a, 0.5), (b, 0.2), (c, 0.4)}, λ_2 = {(a, 0), (b, 0.1), (c,0.3)}, λ_3 = {(a, 0.6), (b, 0.5), (c,0.7)} and λ_4 = {(a, 0.3), (b, 0.4), (c,0.1)}. Let τ = {0, λ_1, λ_2, 1} and σ = {0, λ_3, λ_4, 1}. Define the function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = c and f(c) = b. Then f and f^{-1} are fgspr-continuous but are not fsp-continuous. Therefore f is fgspr-homeomorphism and not fsp-homeomorphism.

Theorem 4: If a function f: (X, τ) → (Y, σ) is fgspr-homeomorphism and X and Y are fuzzy semi preregular $T_{1/2}$ space then f is fsp-homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) is fgspr-homeomorphism. Then f and f^{-1} are fgspr-continuous. To prove that f and f^{-1} are fsp-continuous. Let λ be a fuzzy closed set in Y. Then $f^{-1}(\lambda)$ is a fgspr-closed set in X, since f is fgspr-continuous. Also since X is fuzzy semi preregular $T_{1/2}$ space, $f^{-1}(\lambda)$ is a fsp-closed set in X. Hence f is fsp-continuous.

Now, Let λ be a fuzzy closed set in X. Then $(f^{-1})^{-1}(\lambda)$ = f(λ) is a fgspr-closed set in Y, since f^{-1} is fgspr-continuous. Also since Y is fuzzy semi preregular $T_{1/2}$ space, f(λ) is a fsp-closed set in Y. Hence f^{-1} is fsp-continuous. Thus f is fsp-homeomorphism.

Theorem 5: Let X and Y be fuzzy topological spaces and f: (X, τ) → (Y, σ) be a bijective function. Then the following are equivalent

(i) f^{-1} is fgspr-continuous
(ii) f is fgspr-open
(iii) f is fgspr-closed

Proof: (i) → (ii) Let λ be a fuzzy open set in X. Then $(f^{-1})^{-1}(\lambda)$ = f(λ) is a fgspr-open set in Y, since f^{-1}: (Y, σ) → (X, τ) is fgspr-continuous. Hence f is fgspr-open.

(ii) → (iii) Let λ be a fuzzy closed set in X. Then 1 – λ is a fuzzy open set in X. Since f is fgspr-open, f(1 – λ) is a fgspr-open set in Y. Now f(1 – λ) = 1 – f(λ) is a fgspr-open set in Y. Therefore f(λ) is a fgspr-closed set in Y. Hence f is fgspr-closed.

(iii) → (i) Let λ be a fuzzy closed set in X. Then f(λ) is a fgspr-closed set in Y, since f is fgspr-closed. Now f(λ) = $(f^{-1})^{-1}(\lambda)$ is a fgspr-closed set in Y. Therefore f^{-1}: (Y, σ) → (X, τ) is fgspr-continuous.

Theorem 6: Let X and Y be fuzzy topological spaces and f: (X, τ) → (Y, σ) is a bijective function. Then the following are equivalent

(i) f is fgspr-homeomorphism
(ii) f is fgspr-continuous and fgspr-open
(iii) f is fgspr-continuous and fgspr-closed

Proof: (i) → (ii) Let f is fgspr-homeomorphism. Then f and f^{-1} are fgspr-continuous. To prove that f is fgspr-open. Let λ be a fuzzy open set in X. Then $(f^{-1})^{-1}(\lambda)$ = f(λ) is a fgspr-open set in Y, since f^{-1}: (Y, σ) → (X, τ) is fgspr-continuous. Hence f is fgspr-open.

(ii) → (iii) Let f is fgspr-continuous and fgspr-open. To prove that f is fgspr-closed. Let λ be a fuzzy closed set in X. Then 1 – λ is a fuzzy open set in X. Since f is fgspr-open, f(1 – λ) is a fgspr-open set in Y. Now f(1 – λ) = 1 – f(λ) is a fgspr-open set in Y. Therefore f(λ) is a fgspr-closed set in Y. Hence f is fgspr-closed.

(iii) → (i) Let f is fgspr-continuous and fgspr-closed. To prove that f^{-1} is fgspr-continuous. Let λ be a fuzzy closed set in X. Then f(λ) is a fgspr-closed set in Y, since f is fgspr-closed. Now f(λ) = $(f^{-1})^{-1}(\lambda)$ is a fgspr-closed set in Y. Therefore f^{-1}: (Y, σ) → (X, τ) is fgspr-continuous. Hence f is fgspr-homeomorphism.

Remark 1: The composition of two fgspr-homeomorphism need not be a fgspr-homeomorphism.

Example 4: Let X = {a, b, c}, Y = {a, b, c} and consider the fuzzy sets λ_1 = {(a, 0.5), (b, 0.3), (c, 0.8)}, λ_2 = {(a, 0.8),(b, 1),(c, 0.6)} and λ_3 = {(a, 0.7),(b, 1), (c, 0.5)}. Let τ = {0, λ_1, 1}, σ = {0, λ_2, 1} and η = {0, λ_3, 1}. Define the functions f: (X, τ) → (Y, σ) by f(a) = a, f(b) = c and f(c) = b, g: (Y, σ) → (Z, η) by g(a) = a, g(b) = c and g(c) = b and g o f: (X, τ) → (Z, η) by g o f (a) = a, g o f (b) = b and g o f (c) = c. Then f, f^{-1} and g, g^{-1} are fgspr-continuous but its composition g o f and $(g \circ f)^{-1}$ are not fgspr-continuous. Therefore f and g are fgspr-homeomorphism but its composition g o f is not fgspr-homeomorphism.

Theorem 7: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are fgspr-homeomorphism and Y is fuzzy semi preregular $T^*_{1/2}$ space, then g o f: (X, τ) → (Z, η) is fgspr-homeomorphism.

Proof: To show that g o f and $(g \circ f)^{-1}$ are fgspr-continuous. Let λ be a fuzzy open set in Z. Since g: (Y, σ) → (Z, η) is fgspr-continuous, $g^{-1}(\lambda)$ is a fgspr-open set in Y. Then $g^{-1}(\lambda)$ is a fuzzy open set in Y, as Y is fuzzy semi preregular $T^*_{1/2}$ space. Also, since f is fgspr-continuous, $f^{-1}(g^{-1}(\lambda))$ = $(g \circ f)^{-1}(\lambda)$ is a fgspr-open set in X. Therefore g o f: (X, τ) → (Z, η) is fgspr-continuous.

Again, Let λ be a fuzzy open set in X. Since f^{-1}: (Y, σ) → (X, τ) is fgspr-continuous, $(f^{-1})^{-1}(\lambda) = f(\lambda)$ is a fgspr-open set in Y. Then f(λ) is a fuzzy open set in Y, as Y is fuzzy semi preregular $T^*_{1/2}$ space. Also since g^{-1}: (Z, η) → (Y, σ) is fgspr-continuous, $(g^{-1})^{-1}(f(\lambda)) = g(f(\lambda)) = (g \text{ o } f)(\lambda) = ((g \text{ o } f)^{-1})^{-1}(\lambda)$ is a fgspr-open set in Z. Therefore $(g \text{ o } f)^{-1}$: (Z, η) → (X, τ) is fgspr-continuous. Hence g o f is fgspr-homeomorphism.

Theorem 8: If f: (X, τ) → (Y, σ) is fgspr-homeomorphism and g: (Y, σ) → (Z, η) is f-homeomorphism, then g o f: (X, τ) → (Z, η) is fgspr-continuous.
Proof: To show that g o f is fgspr-continuous. Let λ be a fuzzy open set in Z. Since g: (Y, σ) → (Z, η) is f-continuous, $g^{-1}(\lambda)$ is a fuzzy-open set in Y. Also since f is fgspr-continuous, $f^{-1}(g^{-1}(\lambda)) = (g \text{ o } f)^{-1}(\lambda)$ is a fgspr-open set in X. Therefore g o f: (X, τ) → (Z, η) is fgspr-continuous.

Theorem 9: If f: (X, τ) → (Y, σ) is f-homeomorphism and g: (Y, σ) → (Z, η) is fgspr-homeomorphism, then $(g \text{ o } f)^{-1}$: (Z, η) → (X, τ) is fgspr-continuous.
Proof: To show that $(g \text{ o } f)^{-1}$ is fgspr-continuous. Let λ be a fuzzy open set in X. Since f^{-1}: (Y, σ) → (X, τ) is f-continuous, $(f^{-1})^{-1}(\lambda) = f(\lambda)$ is a fuzzy-open set in Y. Also since g^{-1}: (Z, η) → (Y, σ) is fgspr-continuous, $(g^{-1})^{-1}(f(\lambda)) = g(f(\lambda)) = (g \text{ o } f)(\lambda) = ((g \text{ o } f)^{-1})^{-1}(\lambda)$ is a fgspr-open set in Z. Therefore $(g \text{ o } f)^{-1}$: (Z, η) → (X, τ) is fgspr-continuous.

Theorem 10: If a bijective function f: (X, τ) → (Y, σ) is fgspr-homeomorphism then f[fgspr-cl(λ)] ≤ cl[f(λ)] for every fuzzy set λ in X.
Proof: Let f is fgspr-homeomorphism. Then f and f^{-1} are fgspr-continuous. Let λ be any fuzzy set in X. Now cl[f(λ)] is a fuzzy closed set in Y. As f is fgspr-continuous, f^{-1}(cl[f(λ)]) is a fgspr-closed set in X. λ ≤ f^{-1}(cl[f(λ)]) and so fgspr-cl(λ) ≤ f^{-1}(cl[f(λ)]). Hence f [fgspr-cl(λ)] ≤ cl[f(λ)].

Theorem 11: If a bijective function f: (X, τ) → (Y, σ) is fgspr-homeomorphism then fgspr-cl[f^{-1}(μ)] ≤ f^{-1}[cl(μ)] for every fuzzy set μ in Y.
Proof: Let f is fgspr-homeomorphism. Then f and f^{-1} are fgspr-continuous. Let μ be any fuzzy set in Y. Now cl(μ) is a fuzzy closed set in Y. As f is fgspr-continuous, f^{-1}[cl(μ)] is a fgspr-closed set in X. From Theorem 10, f^{-1}[cl(μ)] ≥ fgspr-cl[f^{-1}[cl(μ)]] ≥ fgspr-cl[f^{-1}(μ)]. Hence fgspr-cl[f^{-1}(μ)] ≤ f^{-1}[cl(μ)].

Theorem 12: If a bijective function f: (X, τ) → (Y, σ) is fgspr-homeomorphism then int[f(λ)] ≤ f[fgspr-int(λ)] for every fuzzy set λ in X.
Proof: Let f is fgspr-homeomorphism. Then f and f^{-1} are fgspr-continuous. Let λ be any fuzzy set in X. Now int[f(λ)] is a fuzzy open set in Y. As f is fgspr-continuous f^{-1}(int[f(λ)]), is a fgspr-open set in X. λ ≥ f^{-1}(int[f(λ)]) and so fgspr-int(λ) ≥ f^{-1}(int[f(λ)]). Hence int[f(λ)] ≤ f[fgspr-int(λ)].

Theorem 13: If a bijective function f: (X, τ) → (Y, σ) is fgspr-homeomorphism then f^{-1}[int(μ)] ≤ fgspr-int[f^{-1}(μ)] for every fuzzy set μ in Y,
Proof: Let f is fgspr-homeomorphism. Then f and f^{-1} are fgspr-continuous. Let μ be any fuzzy set in Y. Now int(μ) is a fuzzy open set in Y. As f is fgspr-continuous, f^{-1}[int(μ)] is a fgspr-open set in X. From Theorem 12, f^{-1}[int(μ)] ≤ fgspr-int(f^{-1}[int(μ)]) ≤ fgspr-int(f^{-1}(μ)). Hence f^{-1}[int(μ)] ≤ fgspr-int[f^{-1}(μ)].

Theorem 14: The set fgspr-homeomorphism (X, τ) is a group under the composition of functions.
Proof: Define a binary operation *: FGSPR-H(X, τ) × FGSPR-H(X, τ) → FGSPR-H(X, τ) by f * g = g o f for all f, g ϵ FGSPR-H(X, τ) and o is the usual operation of composition of functions. From Theorem 7, g o f ϵ FGSPR-H(X, τ). We know that, the composition of functions is associative and the identity map I: (X, τ) → (X, τ) belonging to FGSPR-H(X, τ) serves as the identity element. If f ϵ FGSPR-H(X, τ), then f^{-1} ϵ FGSPR-H(X, τ) such that f o f^{-1}= f^{-1} o f = I and so inverse exists for each element of FGSPR-H(X, τ). Therefore (FGSPR-H(X, τ), o) is a group under the operation of composition of functions.

Theorem 15: Let f: (X, τ) → (Y, σ) be a fgspr-homeomorphism. Then f induces an isomorphism from the group FGSPR-H(X, τ) onto the group FGSPR-H(Y, σ).
Proof: Using the function f, we define a function μ_f **:** FGSPR-H(X, τ) → FGSPR-H(Y, σ) by $\mu_f(h) = f \text{ o } h \text{ o } f^{-1}$ for every h ϵ FGSPR-H(X, τ). Then μ_f is a bijection function. Further, for all h_1, h_2 ϵ FGSPR-H(X, τ), $\mu_f(h_1 \text{ o } h_2) = f \text{ o } (h_1 \text{ o } h_2) \text{ o } f^{-1} = (f \text{ o } h_1 \text{ o } f^{-1}) \text{ o } (f \text{ o } h_2 \text{ o } f^{-1}) = \mu_f(h_1) \text{ o } \mu_f(h_2)$. Therefore μ_f is a homeomorphism and so it is an isomorphism induced by f.

IV. FGSPR*-HOMEOMORPHISM

In this section, some properties of fuzzy generalized semi preregular*-homeomorphism are studied.

Definition 8: Let X and Y be two fuzzy topological spaces. A bijective function f: (X, τ) → (Y, σ) is called fuzzy generalized semi preregular*-homeomorphism (briefly fgspr*-homeomorphism) iff both f and f^{-1} are fgspr-irresolute.

We denote the family of all fgspr*-homeomorphism of a fuzzy topological space (X, τ) onto itself by FGSPR*-H(X, τ).

Example 5: Let X = {a, b, c}, Y = {a, b, c} and consider the fuzzy sets λ_1 = {(a, 0), (b, 0), (c, 1)}, λ_2 = {(a, 0), (b, 1), (c, 1)} and λ_3 = {(a, 1), (b, 1), (c, 0)}. Let τ = {0, λ_1, λ_2, 1} and σ = {0, λ_3, 1}. Define the function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = c and f(c) = b. Then f and f^{-1} are fgspr-irresolute. Therefore f is fgspr*-homeomorphism.

Theorem 16: Every fgspr*-homeomorphism is fgspr-homeomorphism.
Proof: Let f: (X, τ) → (Y, σ) is fgspr*-homeomorphism. Then f and f^{-1} are fgspr-irresolute. Since every fgspr-irresolute is fgspr-continuous, therefore f and f^{-1} are fgspr-continuous. Hence f is fgspr-homeomorphism.

The following example shows that the converse of the above theorem is not true.

Example 6: Let X = {a, b, c}, Y = {a, b, c} and consider the fuzzy sets λ_1 = {(a, 1), (b, 0), (c, 0)}, λ_2 = {(a, 1), (b, 1), (c, 0)} and λ_3 = {(a, 0), (b, 1), (c, 1)}. Let τ = {0, λ_1, λ_2, 1} and σ = {0, λ_3, 1}. Define the function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = c and f(c) = b. Then f and f^{-1} are fgspr-continuous but are not fgspr-irresolute. Therefore f is fgspr-homeomorphism and not fgspr*-homeomorphism.

Theorem 17: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are fgspr*-homeomorphism, then g o f: (X, τ) → (Z, η) is fgspr*-homeomorphism.
Proof: To show that g o f and $(g \circ f)^{-1}$ are fgspr-irresolute. Let λ be a fgspr-open set in Z. Since g: (Y, σ) → (Z, η) is fgspr-irresolute, $g^{-1}(\lambda)$ is a fgspr-open set in Y. Also since f is fgspr-irresolute, $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is a fgspr-open set in X. Therefore g o f: (X, τ) → (Z, η) is fgspr-irresolute.
Again, Let λ be a fgspr-open set in X. Since f^{-1}: (Y, σ) → (X, τ) is fgspr-irresolute, $(f^{-1})^{-1}(\lambda) = f(\lambda)$ is a fgspr-open set in Y. Also since g^{-1}: (Z, η) → (Y, σ) is fgspr-irresolute, $(g^{-1})^{-1}(f(\lambda)) = g(f(\lambda)) = (g \circ f)(\lambda) = ((g \circ f)^{-1})^{-1}(\lambda)$ is a fgspr-open set in Z. Therefore $(g \circ f)^{-1}$: (Z, η) → (X, τ) is fgspr-irresolute. Hence g o f: (X, τ) → (Z, η) is fgspr*-homeomorphism.

Theorem 18: Let X and Y be fuzzy topological spaces and f: (X, τ) → (Y, σ) is a bijective function. Then the following are equivalent
(i) f^{-1} is fgspr-irresolute
(ii) f is fgspr*-open
(iii) f is fgspr*-closed
Proof: (i) → (ii) Let λ be a fgspr-open set in X. Then $(f^{-1})^{-1}(\lambda) = f(\lambda)$ is a fgspr-open set in Y, since f^{-1}: (Y, σ) → (X, τ) is fgspr-continuous. Hence f is fgspr*-open.

(ii) → (iii) Let λ be a fgspr-closed set in X. Then 1 – λ is a fgspr-open set in X. Since f is fgspr*-open, f(1 – λ) is a fgspr-open set in Y. Now f(1 – λ) = 1 – f(λ) is a fgspr-open set in Y. Therefore f(λ) is a fgspr-closed set in Y. Hence f is fgspr*-closed.

(iii) → (i) Let λ be a fgspr-closed set in X. Then f(λ) is a fgspr-closed set in Y, since f is fgspr*-closed. Now f(λ) = $(f^{-1})^{-1}(\lambda)$ is a fgspr-closed set in Y. Therefore f^{-1}: (Y, σ) → (X, τ) is fgspr-irresolute.

Theorem 19: Let X and Y be fuzzy topological spaces and f: (X, τ) → (Y, σ) is a bijective function. Then the following are equivalent
(iv) f is fgspr*-homeomorphism
(v) f is fgspr-irresolute and fgspr*-open
(vi) f is fgspr-irresolute and fgspr*-closed
Proof: (i) → (ii) Let f is fgspr*-homeomorphism. Then f and f^{-1} are fgspr-irresolute. To prove that f is fgspr*-open. Let λ be a fgspr-open set in X. Then $(f^{-1})^{-1}(\lambda) = f(\lambda)$ is a fgspr-open set in Y, since f^{-1}: (Y, σ) → (X, τ) is

fgspr-irresolute. Hence f is fgspr*-open.

(ii) → (iii) Let f is fgspr-irresolute and fgspr*-open. To prove that f is a fgspr*-closed map. Let λ be a fgspr-closed set in X. Then 1 – λ is a fgspr-open set in X. Since f is fgspr*-open, f(1 – λ) is a fgspr-open set in Y. Now f(1 – λ) = 1 – f(λ) is a fgspr-open set in Y. Therefore f(λ) is a fgspr-closed set in Y. Hence f is fgspr*-closed.

(iii) → (i) Let f is fgspr-irresolute and fgspr*-closed. To prove that f^{-1} is fgspr-irresolute. Let λ be a fgspr-closed set in X. Then f(λ) is a fgspr-closed set in Y, since f is fgspr*-closed. Now f(λ) = $(f^{-1})^{-1}(\lambda)$ is a fgspr-closed set in Y. Therefore f^{-1}: (Y, σ) → (X, τ) is fgspr-irresolute. Hence f is fgspr*-homeomorphism.

Theorem 20: The set fgspr*-homeomorphism (X, τ) is a group under the composition of functions.
Proof: Define a binary operation *: FGSPR*-H(X, τ) × FGSPR*-H(X, τ) → FGSPR*-H(X, τ) by f * g = g o f for all f, g ϵ FGSPR*-H(X, τ) and o is the usual operation of composition of functions. From Theorem 17, g o f ϵ FGSPR*-H(X, τ). We know that, the composition of functions is associative and the identity map I: (X, τ) → (X, τ) belonging to FGSPR*-H(X, τ) serves as the identity element. If f ϵ FGSPR*-H(X, τ), then f^{-1} ϵ FGSPR*-H(X, τ) such that f o f^{-1}= f^{-1} o f = I and so inverse exists for each element of FGSPR*-H(X, τ). Therefore (FGSPR*-H(X, τ), o) is a group under the operation of composition of functions.

Theorem 21: Let f: (X, τ) → (Y, σ) be a fgspr*-homeomorphism. Then f induces an isomorphism from the group FGSPR*-H(X, τ) onto the group FGSPR*-H(Y, σ).
Proof: Using the function f, we define a function μ_f: FGSPR*-H(X, τ) → FGSPR*-H(Y, σ) by μ_f(h) = f o h o f^{-1} for every h ϵ FGSPR*-H(X, τ). Then μ_f is a bijection function. Further, for all h_1, h_2 ϵ FGSPR*-H(X, τ), $\mu_f(h_1 \text{ o } h_2)$ = f o (h_1 o h_2) o f^{-1} = (f o h_1 o f^{-1}) o (f o h_2 o f^{-1}) = $\mu_f(h_1)$ o $\mu_f(h_2)$. Therefore μ_f is a homeomorphism and so it is an isomorphism induced by f.

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